OCR
RECOGNISING ACHIEVEMENT

Friday 22 June 2012 – Afternoon

A2 GCE MATHEMATICS

4729 Mechanics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4729
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A particle, of mass 0.8 kg, moves along a smooth horizontal surface. It hits a vertical wall, which is at right angles to the direction of motion of the particle, and rebounds. The speed of the particle as it hits the wall is $4\,\mathrm{m\,s^{-1}}$ and the coefficient of restitution between the particle and the wall is 0.3. Find

(i) the impulse that the wall exerts on the particle, **[3]**

(ii) the kinetic energy lost in the impact. **[2]**

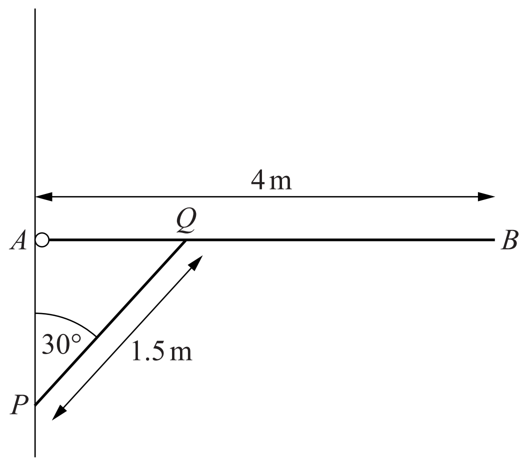
2 A car of mass 1600 kg moves along a straight horizontal road. The resistance to the motion of the car has constant magnitude 800 N and the car's engine is working at a constant rate of 20 kW.

(i) Find the acceleration of the car at an instant when the car's speed is $20\,\mathrm{m\,s^{-1}}$. **[4]**

The car now moves up a hill inclined at 4° to the horizontal. The car's engine continues to work at 20 kW and the magnitude of the resistance to motion remains at 800 N.

(ii) Find the greatest steady speed at which the car can move up the hill. **[4]**

3



A uniform beam *AB* of mass 15 kg and length 4 m is freely hinged to a vertical wall at *A*. The beam is held in equilibrium in a horizontal position by a light rod *PQ* of length 1.5 m. *P* is fixed to the wall vertically below *A* and *PQ* makes an angle of 30° with the vertical (see diagram). The force exerted on the beam at *Q* by the rod is in the direction *PQ*. Find

(i) the magnitude of the force exerted on the beam at *Q*, **[3]**

(ii) the magnitude and direction of the force exerted on the beam at *A*. **[6]**

4 A boy throws a small ball at a vertical wall. The ball is thrown horizontally, from a point O, at a speed of $14.4\,\text{m}\,\text{s}^{-1}$ and it hits the wall at a point which is 0.2 m below the level of O.

(i) Find the horizontal distance from O to the wall. **[4]**

The boy now moves so that he is 6 m from the wall. He throws the ball at an angle of 15° above the horizontal. The ball again hits the wall at a point which is 0.2 m below the level from which it was thrown.

(ii) Find the speed at which the ball was thrown. **[6]**

5 A particle P, of mass 2 kg, is attached to fixed points A and B by light inextensible strings, each of length 2 m. A and B are 3.2 m apart with A vertically above B. The particle P moves in a horizontal circle with centre at the mid-point of AB.

(i) Find the tension in each string when the angular speed of P is $4\,\text{rad}\,\text{s}^{-1}$. **[7]**

(ii) Find the least possible speed of P. **[6]**

6 Three particles A, B and C are in a straight line on a smooth horizontal surface. The particles have masses 0.2 kg, 0.4 kg and 0.6 kg respectively. B is at rest. A is projected towards B with a speed of $1.8\,\text{m}\,\text{s}^{-1}$ and collides with B. The coefficient of restitution between A and B is $\frac{1}{3}$.

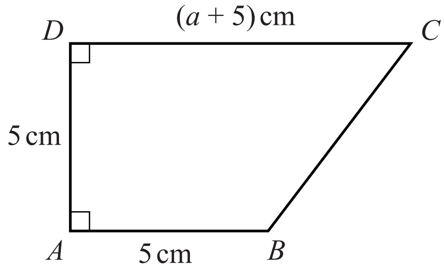
(i) Show that the speed of B after the collision is $0.8\,\text{m}\,\text{s}^{-1}$ and find the speed of A after the collision. **[6]**

C is moving with speed $0.2\,\text{m}\,\text{s}^{-1}$ in the same direction as B. Particle B subsequently collides with C. The coefficient of restitution between B and C is e.

(ii) Find the set of values for e such that B does not collide again with A. **[7]**

[Question 7 is printed overleaf.]

7



The diagram shows the cross-section through the centre of mass of a uniform solid prism. The cross-section is a trapezium *ABCD* with *AB* and *CD* perpendicular to *AD*. The lengths of *AB* and *AD* are each 5 cm and the length of *CD* is $(a + 5)$ cm.

(i) Show the distance of the centre of mass of the prism from *AD* is

$$\frac{a^2 + 15a + 75}{3(a + 10)} \text{ cm.}$$ **[5]**

The prism is placed with the face containing *AB* in contact with a horizontal surface.

(ii) Find the greatest value of a for which the prism does not topple. **[3]**

The prism is now placed on an inclined plane which makes an angle $\theta°$ with the horizontal. *AB* lies along a line of greatest slope with *B* higher than *A*.

(iii) Using the value for a found in part **(ii)**, and assuming the prism does not slip down the plane, find the greatest value of θ for which the prism does not topple. **[6]**

Question			Answer	Marks	Guidance
1	(i)		Speed = 1.2 ms^{-1} Impulse = 0.8 × ± (4 – –1.2) ±4.16 Ns	B1 M1 A1 **[3]**	May be seen anywhere, even in **(ii)**; allow -1.2 Difference between momenta, allow 0.8 × ± (4 – 1.2)
1	(ii)		KE lost = ½ × 0.8 × (4^2 –$(\pm1.2)^2$) 5.82(4) J	M1 A1 **[2]**	 Allow -5.82(4)
2	(i)		Driving Force = 20000/20 (= 1000) 20000/20 – 800 = 1600*a* *a* = 0.125 ms^{-2}	B1 M1 A1 A1 **[4]**	 Attempt at N2L with 3 terms. Signs may not be correct at this stage. Using their 20000/20, but not 20000 Allow $\frac{1}{8}$
2	(ii)		20000/*v* DF – 800 – 1600*g*sin4 = 0 *v* = 10.6 ms^{-1}	B1 M1 A1 A1 **[4]**	 3 terms with attempt at resolving weight; *g* can be omitted at this stage; if F = ….. then F = 0 somewhere to award M aef
3	(i)		 *T*cos30 × 1.5sin 30 = 15*g* × 2 *T* = 453	M1 A1 A1 **[3]**	Attempt at moments about *A, g* can be omitted at this stage
3	(ii)		$X = T_c\sin30$ (=226) $Y + T_c\cos30 = 15g$ *R* = √(226^2 + 245^2) or tan*θ* = 245/226 *R* = 334 *θ* = 47.3 below horizontal (to the left)	B1ft M1 A1ft M1 A1 A1 **[6]**	Using their value *T* or taking moments about *P* Attempt to resolve vertically or taking appropriate moments Using their value *T*; expect *Y* = -245 or better Either or both of these equations can be replaced with moments about an appropriate point eg *P*, *Q*, *B*, c of m of beam. Any relevant angle Allow 333 Allow 47.2, 42.7 to the downward vertical SC: If *392* in (i) leading to *Y=±245* only in (ii) max M1A1

Question			Answer	Marks	Guidance
4	(i)		$\frac{1}{2} \times 9.8 \times t^2 = 0.2$ $t = 0.2(02)$ $s = 14.4 \times t_c$ $s = 2.91$ m	M1 A1 M1 A1 **[4]**	Using SUVAT to find t, consistent signs for *g* and *0.2* aef Using their value of *t*
		OR	Use equation of trajectory $-0.2 = x\tan 0 - gx^2 \sec^2 0/(2\text{x}14.4^2)$ Solve quadratic for x $x = 2.91$	M1 A1 M1 A1 **[4]**	B1 for correct equation of the trajectory seen anywhere but award in part (ii) unless different method seen; consistent signs for *g* and *0.2*
4	(ii)		 $U\sin 15 \times t - \frac{1}{2} \times 9.8 \times t^2 = -0.2$ $U\cos 15 \times t = 6$ Eliminate t Attempt to solve to find U $U = 10.2$ ms^{-1}	*M1 A1 B1 Dep*M1 Dep*M1 A1 **[6]**	Using s = $ut + \frac{1}{2}at^2$ with $s = \pm 0.2$ and $a = \pm g$ Eliminate *U* Attempt to solve to find *t(=0.607)*
		OR	$y = x\tan\theta - gx^2 \sec^2 \theta/2U^2$ Substitute values for y, x, θ $-0.2 = 6\tan 15 - g.6^2 \sec^2 15/2U^2$ Attempt to solve for U $U = 10.2$ ms^{-1}	*B1 Dep*M1 A1 Dep*M2 A1 **[6]**	

Question			Answer	Marks	Guidance
5	(i)		$\sin\theta = 0.8$ or $\cos\theta = 0.6$ or $\tan\theta = 4/3$ or $\theta = 53.1$	B1	θ is angle AP makes with horizontal
				*M1	Attempt to resolve horizontally and use N2L with a version of acceleration, not just a. Allow $T_A = T_B$ for M1 only.
			$T_A\cos\theta + T_B\cos\theta = 2 \times 1.2 \times 4^2$	A1	Use their θ
				*M1	Attempt to resolve vertically
			$T_A\sin\theta = T_B\sin\theta + 2g$	A1	Use their θ
			Solve simultaneously to get at least T_A or T_B	Dep*M1	
			$T_A = 44.25$ and $T_B = 19.75$	A1	For both. Allow 44.2, 44.3, 19.7, 19.8
				[7]	
5	(ii)		$T_B = 0$	B1	May be implied
				*M1	Attempt to resolve horizontally and use N2L with a version of acceleration, not just a
			$T_A\cos\theta = 2v^2/1.2$	A1	Use their θ
			$T_A\sin\theta = 2g$	B1	Use their θ
			Solve for v or ω	Dep*M1	
			$v = 2.97$	A1	
				[6]	
6	(i)			*M1	Attempt at conservation of momentum
			$0.2 \times 1.8 = 0.2v_A + 0.4v_B$	A1	
				*M1	Attempt at restitution
			$v_B - v_A = ⅓ \times 1.8$	A1	aef
			Solve for v_A or v_B	Dep*M1	
			$v_B = 0.8$ m s^{-1} and $v_A = 0.2$ m s^{-1} **AG**	A1	
				[6]	

Question			Answer	Marks	Guidance
6	(ii)		$0.4 \times 0.8 + 0.6 \times 0.2 = 0.4v_{B'} + 0.6v_C$ $v_C - v_{B'} = e(0.8 - 0.2)$ Use two relevant equations to eliminate v_C State $v_{B'} \geq 0.2$ Set up (in)equality in e and their v_A $0.44 - 0.36e \geq 0.2$ or $0.44 - 0.36e = 0.2$ $e \leq 2/3$ or 0.667	B1 B1 *M1 B1 dep*M1 A1 A1 **[7]**	 aef soi, Allow $v_{B'} > 0.2$ Condone incorrect inequality sign for M1 only Allow $0.44 - 0.36e > 0.2$
		OR	$0.4 \times 0.8 + 0.6 \times 0.2 = 0.4v_{B'} + 0.6v_C$ $v_C - v_{B'} = e(0.8 - 0.2)$ State $v_{B'} \geq 0.2$ Sub $v_{B'}$ in momentum equation & solve for v_C (v_C =) 0.6 Set up (in)equality in e and their v_A $e \leq 2/3$ or 0.667	B1 B1 B1 *M1 A1 dep*M1 A1 **[7]**	 aef soi, Allow $v_{B'} > 0.2$ eg $0.6 - e(0.8 - 0.2) \geq 0.2$, Condone incorrect inequality sign for M1 only

Question			Answer	Marks	Guidance
7	(i)		⅓ a	B1	Centre of mass of triangle
				M1	Table of values idea, using any fixed axis
			$(25 + 2.5a)x_G$	A1	
			$= 25 \times 2.5 + 2.5a \times (5 + ⅓ a)$	A1	Relative to the axis they are using
			$x_G = \dfrac{a^2 + 15a + 75}{3(a+10)}$ **AG**	A1	
				[5]	
7	(ii)		$\dfrac{a^2 + 15a + 75}{3(a+10)} = 5$	*M1	Substitute x_G as 5
			Solving for a	dep*M1	
			$a = 8.66$ or $5\sqrt{3}$	A1	$a \leq 8.66$
				[3]	
7	(iii)			*M1	Method to find centre of mass from AB (or CD) with or without a substituted.
			$(25 + 2.5a)y_G = 25 \times 2.5 + 2.5a \times (⅔ \text{ x } 5)$	A1ft	
			$y_G = \dfrac{10a + 75}{3(a+10)}$ or 2.89	A1ft	ft their a from (ii), from CD y_G=2.11
			$\tan\theta = x_G / y_G$	dep*M1	Using trig to find an appropriate angle, eg complement of θ.
			$= 5 / y_G$	A1ft	ft their a from (ii), but not an incorrect y_G
			$\theta = 60$	A1	$\theta \leq 60$ (anything that rounds to 60)
				[6]	